

*A slide from exercise class 5:*

**Exercise 8.19.**

For each of the following languages, find an unrestricted grammar that generates the language.

**c.**  $\{sss \mid s \in \{a, b\}^*\}$

**d.**  $\{ss^r s \mid s \in \{a, b\}^*\}$

**Exercise 8.34.**

**(b)** Suppose  $G_1 = (V_1, \Sigma, S_1, P_1)$  is an unrestricted grammar generating  $L_1$ .

Using  $G_1$ , describe an unrestricted grammar generating  $L_1^*$ .

### **Exercise 9.1.**

Show that the relation  $\leq$  on the set of decision problems is reflexive and transitive.

Give an example to show that it is not symmetric.

### Exercise 9.5.

*Fermat's last theorem*, until recently one of the most famous unproved statements in mathematics, asserts that there are no integer solutions  $(x, y, z, n)$  to the equation  $x^n + y^n = z^n$  satisfying  $x, y > 0$  and  $n > 2$ .

Ignoring the fact that the theorem has now been proved, explain how a solution to the halting problem would allow you to determine the truth or falsity of the statement.

*A slide from lecture 6:*

*Accepts:* Given a TM  $T$  and a string  $x$ , is  $x \in L(T)$  ?

Instances are ...

*Halts:* Given a TM  $T$  and a string  $x$ , does  $T$  halt on input  $x$  ?

Instances are ...

*Self-Accepting:* Given a TM  $T$ , does  $T$  accept the string  $e(T)$ ?

Instances are ...

Now fix a TM  $T$ :

*$T$ -Accepts:* Given a string  $x$ , does  $T$  accept  $x$  ?

Instances are ...

Decidable or undecidable ? (cf. **Exercise 9.7.**)

## Exercise 9.7.

As discussed at the beginning of Section 9.3, there is at least one TM  $T$  such that the decision problem

“Given  $w$ , does  $T$  accept  $w$  ?”

is unsolvable.

Show that every TM accepting a nonrecursive language has this property.

*A slide from lecture 6:*

Reduction from *Accepts* to *Accepts- $\Lambda$* .

Instance of *Accepts* is  $(T_1, x)$  for TM  $T_1$  and string  $x$ .

Instance of *Accepts- $\Lambda$*  is TM  $T_2$ .

$T_2 = F(T_1, x) =$

$Write(x) \rightarrow T_1$

$T_2$  accepts  $\Lambda$ , if and only if  $T_1$  accepts  $x$ .

### Exercise 9.8.

Show that for every  $w \in \Sigma^*$ , the problem *Accepts* can be reduced to the problem:

Given a TM  $T$ , does  $T$  accept  $w$ ?

(This shows that, just as *Accepts- $\Lambda$*  is unsolvable, so is *Accepts- $w$* , for every  $w$ .)



*Accepts- $\Lambda$* : Given a TM  $T$ , is  $\Lambda \in L(T)$  ?

**Exercise 9.9.**

Construct a reduction from *Accepts- $\Lambda$*  to *Accepts- $\{\Lambda\}$* :

Given a TM  $T$ , is  $L(T) = \{\Lambda\}$  ?