Exercise 8.17.

In each case below, describe the language generated by the unrestricted grammar with the given productions. The symbols a, b, and c are terminals, and all other symbols are variables.

a.

$S \rightarrow ABCS \mid ABC$

$AB \rightarrow BA \quad AC \rightarrow CA \quad BC \rightarrow CB$

$BA \rightarrow AB \quad CA \rightarrow AC \quad CB \rightarrow BC$

$A \rightarrow a \quad B \rightarrow b \quad C \rightarrow c$

Exercise 8.17.

In each case below, describe the language generated by the unrestricted grammar with the given productions. The symbols a, b, and c are terminals, and all other symbols are variables.

b.

$$S \rightarrow LaR \quad L \rightarrow LD \mid LT \mid \wedge \quad Da \rightarrow aaD \quad Ta \rightarrow aaaT$$

$$DR \rightarrow R \quad TR \rightarrow R \quad R \rightarrow \wedge$$

Exercise 8.17.

In each case below, describe the language generated by the un-restricted grammar with the given productions. The symbols a, b, and c are terminals, and all other symbols are variables.

c.

$$S \to LaMR \qquad L \to LT \mid E$$

$$Ta \to aT \qquad TM \to aaMT \qquad TR \to aMR$$

$$Ea \to aE \qquad EM \to E \qquad ER \to \Lambda$$

Exercise 8.18.

Consider the unrestricted grammar with the following productions.

$S \to TD_1D_2 \quad T \to ABCT \mid \Lambda$

$AB \to BA \quad BA \to AB \quad CA \to AC \quad CB \to BC$

$CD_1 \to D_1C \quad CD_2 \to D_2a \quad BD_1 \to D_1b$

$A \to a \quad D_1 \to \Lambda \quad D_2 \to \Lambda$

a. Describe the language generated by this grammar.

b. Find a single production that could be substituted for $BD_1 \to D_1b$ so that the resulting language would be

$$\{xa^n \mid \ n \geq 0, \ |x| = 2n, \text{ and } n_a(x) = n_b(x) = n\}$$

Exercise 8.19.

For each of the following languages, find an unrestricted grammar that generates the language.

a. $\{a^n b^n a^n b^n \mid n \geq 0\}$

Exercise 8.19.

For each of the following languages, find an unrestricted grammar that generates the language.

c. $\{sss \mid s \in \{a, b\}^*\}$

d. $\{ss^r s \mid s \in \{a, b\}^*\}$

Exercise 8.20.

For each of the following languages, find an unrestricted grammar that generates the language.

a. $\{x \in \{a, b, c\}^* \mid n_a(x) < n_b(x) \text{ and } n_a(x) < n_c(x)\}$

c. $\{a^n \mid n = j(j+1)/2 \text{ for some } j \geq 1\}$
(Suggestion: if a string has j groups of a's, the ith group containing i a's, then you can create $j + 1$ groups by adding an a to each of the j groups and adding a single extra a at the beginning.)

Exercise 8.21.

Suppose G is an unrestricted grammar with start symbol T that generate the language $L \subseteq \{a, b\}^*$. In each part below, another unrestricted grammar is described. Say (in terms of L) what language it generates.

a. The grammar containing all the variables and all the productions of G, two additional variables S (the start variable) and E, and the additional productions

$$S \to ET \quad E \to \wedge \quad Ea \to E \quad Eb \to E$$

Exercise 8.21.

Suppose G is an unrestricted grammar with start symbol T that generate the language $L \subseteq \{a, b\}^*$. In each part below, another unrestricted grammar is described. Say (in terms of L) what language it generates.

b. The grammar containing all the variables and all the productions of G, four additional variables S (the start variable), F, R, and E, and the additional productions

$$S \to FTR \quad Fa \to aF \quad Fb \to bF \quad F \to E$$
$$Ea \to E \quad Eb \to E \quad ER \to \wedge$$

Exercise.

For the Turing machine below, give all productions of the second type (to simulate the computation of the Turing machine) that result from the construction in the proof of Theorem 8.14.
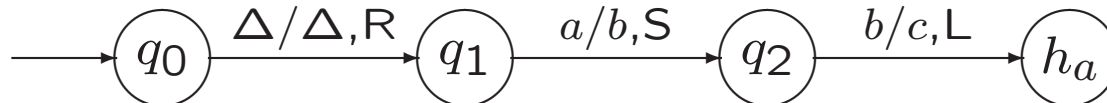
Exercise 8.22.

Figure 7.6 shows the transition diagram for a TM accepting $XX = \{xx \mid x \in \{a, b\}^*\}$.

In the grammar obtained from this TM as in the proof of Theorem 8.14, give a derivation for the string $abab$.

Exercise 8.27.

Show that if L is any recursively enumerable language, then L can be generated by a grammar in which the left side of every production is a string of one or more variables.