Exercise 9.1.

Show that the relation $\leq$ on the set of decision problems is reflexive and transitive.

Give an example to show that it is not symmetric.

Exercise 9.5.

Fermat's last theorem, until recently one of the most famous unproved statements in mathematics, asserts that there are no integer solutions (x, y, z, n) to the equation $x^n + y^n = z^n$ satisfying $x, y > 0$ and $n > 2$.

Ignoring the fact that the theorem has now been proved, explain how a solution to the halting problem would allow you to determine the truth or falsity of the statement.

A slide from lecture 6:

Accepts: Given a TM T and a string x, is $x \in L(T)$?
Instances are . . .

Halts: Given a TM T and a string x, does T halt on input x ?
Instances are . . .

Self-Accepting: Given a TM T, does T accept the string $e(T)$?
Instances are . . .

Now fix a TM T:
T-*Accepts*: Given a string x, does T accept x ?
Instances are . . .
Decidable or undecidable ? (cf. **Exercise 9.7.**)

Exercise 9.7.

As discussed at the beginning of Section 9.3, there is at least one TM T such that the decision problem

"Given w, does T accept w ?"

is unsolvable.

Show that every TM accepting a nonrecursive language has this property.

A slide from lecture 6:

Reduction from *Accepts* to *Accepts*-Λ.

Instance of *Accepts* is (T_1, x) for TM T_1 and string x.
Instance of *Accepts*-Λ is TM T_2.

$T_2 = F(T_1, x) =$

$$\textit{Write}(x) \to T_1$$

T_2 accepts Λ, if and only if T_1 accepts x.

Exercise 9.8.

Show that for every $w \in \Sigma^*$, the problem *Accepts* can be reduced to the problem:

Given a TM T, does T accept w?

(This shows that, just as *Accepts*-Λ is unsolvable, so is *Accepts*$-w$, for every w.)

Accepts-Λ: Given a TM T, is $\Lambda \in L(T)$?

Exercise 9.9.

Construct a reduction from *Accepts*-Λ to *Accepts*-$\{\Lambda\}$:

Given a TM T, is $L(T) = \{\Lambda\}$?