

Exercise 8.17.

In each case below, describe the language generated by the unrestricted grammar with the given productions. The symbols a , b , and c are terminals, and all other symbols are variables.

a.

$$S \rightarrow ABCS \mid ABC$$

$$AB \rightarrow BA \quad AC \rightarrow CA \quad BC \rightarrow CB$$

$$BA \rightarrow AB \quad CA \rightarrow AC \quad CB \rightarrow BC$$

$$A \rightarrow a \quad B \rightarrow b \quad C \rightarrow c$$

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In each case below, describe the language generated by the unrestricted grammar with the given productions. The symbols a , b , and c are terminals, and all other symbols are variables.

b.

$$S \rightarrow LaR \quad L \rightarrow LD \mid LT \mid \Lambda \quad Da \rightarrow aaD \quad Ta \rightarrow aaaT$$

$$DR \rightarrow R \quad TR \rightarrow R \quad R \rightarrow \Lambda$$

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In each case below, describe the language generated by the unrestricted grammar with the given productions. The symbols a , b , and c are terminals, and all other symbols are variables.

c.

$$S \rightarrow LaMR \quad L \rightarrow LT \mid E$$

$$Ta \rightarrow aT \quad TM \rightarrow aaMT \quad TR \rightarrow aMR$$

$$Ea \rightarrow aE \quad EM \rightarrow E \quad ER \rightarrow \Lambda$$

Exercise 8.18.

Consider the unrestricted grammar with the following productions.

$$S \rightarrow TD_1D_2 \quad T \rightarrow ABCT \mid \Lambda$$

$$AB \rightarrow BA \quad BA \rightarrow AB \quad CA \rightarrow AC \quad CB \rightarrow BC$$

$$CD_1 \rightarrow D_1C \quad CD_2 \rightarrow D_2a \quad BD_1 \rightarrow D_1b$$

$$A \rightarrow a \quad D_1 \rightarrow \Lambda \quad D_2 \rightarrow \Lambda$$

- a. Describe the language generated by this grammar.
- b. Find a single production that could be substituted for $BD_1 \rightarrow D_1b$ so that the resulting language would be

$$\{xa^n \mid n \geq 0, |x| = 2n, \text{ and } n_a(x) = n_b(x) = n\}$$

Exercise 8.19.

For each of the following languages, find an unrestricted grammar that generates the language.

a. $\{a^n b^n a^n b^n \mid n \geq 0\}$

Exercise 8.19.

For each of the following languages, find an unrestricted grammar that generates the language.

c. $\{sss \mid s \in \{a, b\}^*\}$

d. $\{ss^r s \mid s \in \{a, b\}^*\}$

Exercise 8.20.

For each of the following languages, find an unrestricted grammar that generates the language.

a. $\{x \in \{a, b, c\}^* \mid n_a(x) < n_b(x) \text{ and } n_a(x) < n_c(x)\}$

c. $\{a^n \mid n = j(j + 1)/2 \text{ for some } j \geq 1\}$

(Suggestion: if a string has j groups of a 's, the i th group containing i a 's, then you can create $j + 1$ groups by adding an a to each of the j groups and adding a single extra a at the beginning.)

Exercise 8.21.

Suppose G is an unrestricted grammar with start symbol T that generate the language $L \subseteq \{a, b\}^*$. In each part below, another unrestricted grammar is described. Say (in terms of L) what language it generates.

a. The grammar containing all the variables and all the productions of G , two additional variables S (the start variable) and E , and the additional productions

$$S \rightarrow ET \quad E \rightarrow \Lambda \quad Ea \rightarrow E \quad Eb \rightarrow E$$

Exercise 8.21.

Suppose G is an unrestricted grammar with start symbol T that generate the language $L \subseteq \{a, b\}^*$. In each part below, another unrestricted grammar is described. Say (in terms of L) what language it generates.

b. The grammar containing all the variables and all the productions of G , four additional variables S (the start variable), F , R , and E , and the additional productions

$$\begin{aligned} S \rightarrow FTR \quad Fa \rightarrow aF \quad Fb \rightarrow bF \quad F \rightarrow E \\ Ea \rightarrow E \quad Eb \rightarrow E \quad ER \rightarrow \Lambda \end{aligned}$$

Exercise 8.27.

Show that if L is any recursively enumerable language, then L can be generated by a grammar in which the left side of every production is a string of one or more variables.