Exercise 7.14.

Draw the *Insert*(σ) TM, which changes the tape contents from $y\underline{z}$ to $y\underline{\sigma}z$.
Here $y \in (\Gamma \cup \{\Delta\})^*$, $\sigma \in \Gamma \cup \{\Delta\}$, and $z \in \Gamma^*$.
You may assume that $\Gamma = \{a, b\}$.

Exercise 7.13.

Suppose T is a TM that accepts every input. We might like to construct a TM R_T such that for every input string x, R_T halts in the accepting state with exactly the same tape contents as when T halts on input x, but with the tape head positioned at the rightmost nonblank symbol on the tape.

Show that there is no fixed TM T_0 such that $R_T = TT_0$ for every T. (In other words, there is no TM capable of executing the instruction "move the tape head to the rightmost nonblank tape symbol" in every possible situation.)

Suggestion: Assume there is such a TM T_0, and try to find two other TMs T_1 and T_2 such that if $R_{T_1} = T_1T_0$ then R_{T_2} cannot be T_2T_0.

Assume that the tape contains at least one nonblank symbol, when T halts.

Exercise 7.18.

The TM shown in Figure 7.38 computes a function f from $\{a, b\}^*$ to $\{a, b\}^*$. For any string $x \in \{a, b\}^*$, describe the string $f(x)$.

Exercise 7.19.

Suppose TMs T_1 and T_2 compute the functions f_1 and f_2 from $\mathbb{N}$ to $\mathbb{N}$, respectively.

Describe how to construct a TM to compute the function f_1+f_2.

Exercise 7.20.

Draw a transition diagram for a TM with input alphabet $\{0, 1\}$ that interprets the input string as the binary representation of a nonnegative integer and adds 1 to it.

You may assume that the input string is not empty.

Exercise.

Construct a 2-tape Turing machine T that has as input two strings w_1 and w_2 from $\{a, b\}^*$ (both on the first tape, separated by a single blank, as usual), and that checks in linear time whether or not w_2 is an anagram of w_1 (a rearrangement of the letters). If so, then T should accept, otherwise, it should reject.

Hint: in order to check if w_2 *is an anagram of* w_1*, you might look at the number of occurrences of letters in* w_1 *and* w_2*.*

Exercise 7.23.

Draw a transition diagram for a three-tape TM that works as follows:

starting in the configuration $(q_0, \underline{\Delta}x, \underline{\Delta}y, \underline{\Delta})$,

where x and y are nonempty strings of 0's and 1's of the same length,

it halts in the configuration $(h_a, \underline{\Delta}x, \underline{\Delta}y, \underline{\Delta}z)$,

where z is the string obtained by interpreting x and y as binary representations and adding them.

Use transitions of the following form:

$$p \xrightarrow{(X_1, X_2, X_3)/(Y_1, Y_2, Y_3), (D_1, D_2, D_3)} q$$

Exercise 7.25.

We can consider a TM with a *doubly infinite* tape, by allowing the numbers of the tape squares to be negative as well as positive. In most respects the rules for such a TM are the same as for an ordinary one, except that now when we refer to the configuration $xq\sigma y$, including the initial configuration corresponding to some input string, there is no assumption about exactly where on the tape the strings and the tape head are.

Draw a transition diagram for a TM with a doubly infinite tape that does the following: If it begins with the tape blank except for a single a somewhere on it, it halts in the accepting state with the head on the square with the a.