Computability

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college 3, 18 februari 2021

7. Turing Machines

7.4. Combining Turing Machines

7.5. Multitape Turing Machines

7.7. Nondeterministic Turing Machines

7.4. Combining Turing Machines
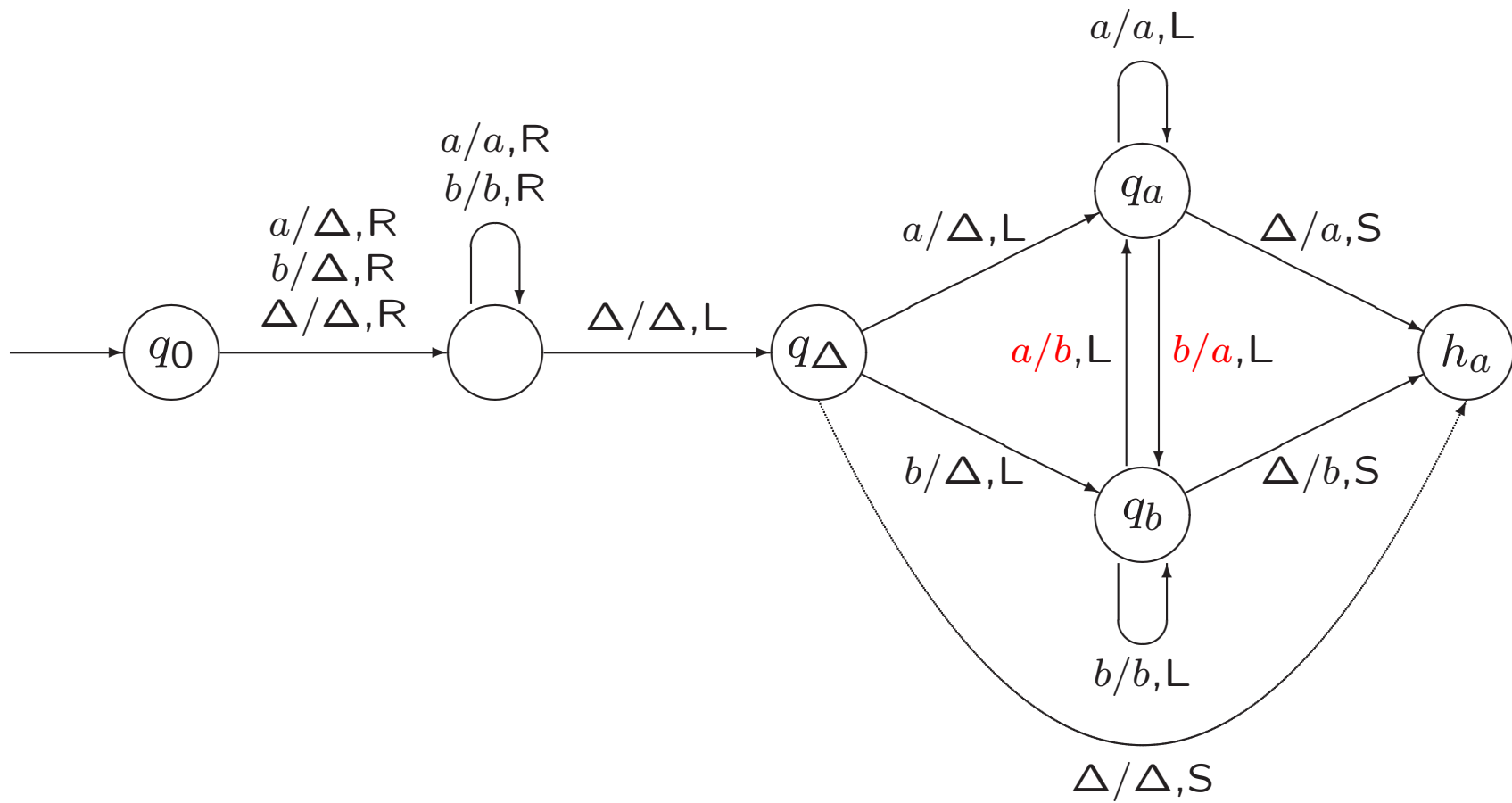
Example.

A TM for $f(x) = a^{n_a(x)}$

$x = aababba$

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$$
\begin{array}{l}
\underline{\triangle}\ a\ a\ b\ a\ b\ b\ a \\
\triangle\ a\ a\ \underline{\triangle}\ a\ b\ b\ a \\
\triangle\ a\ a\ \underline{a}\ b\ b\ a\ \triangle \\
\triangle\ a\ a\ a\ \underline{\triangle}\ b\ a\ \triangle \\
\triangle\ a\ a\ a\ \underline{b}\ a\ \triangle\triangle \\
\triangle\ a\ a\ a\ \underline{\triangle}\ a\ \triangle\triangle \\
\triangle\ a\ a\ a\ \underline{a}\ \triangle\triangle\triangle \\
\underline{\triangle}\ a\ a\ a\ a\ \triangle\triangle\triangle
\end{array}
$$

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Example 7.20. Inserting and Deleting a Symbol

Delete: from $y\underline{\sigma}z$ to $y\underline{z}$

Insert(σ): from $y\underline{z}$ to $y\underline{\sigma}z$

N.B.: z does not contain blanks

TM T_1 computes f

TM T_2 computes g

TM T_1T_2 computes ...

$$T_1 \longrightarrow T_2$$

Example 7.17. Finding the Next Blank or the Previous Blank

NB

PB

Example 7.18. Copying a String

Copy: from $\underline{\triangle}x$ to $\underline{\triangle}x\triangle x$

$x = abaa$
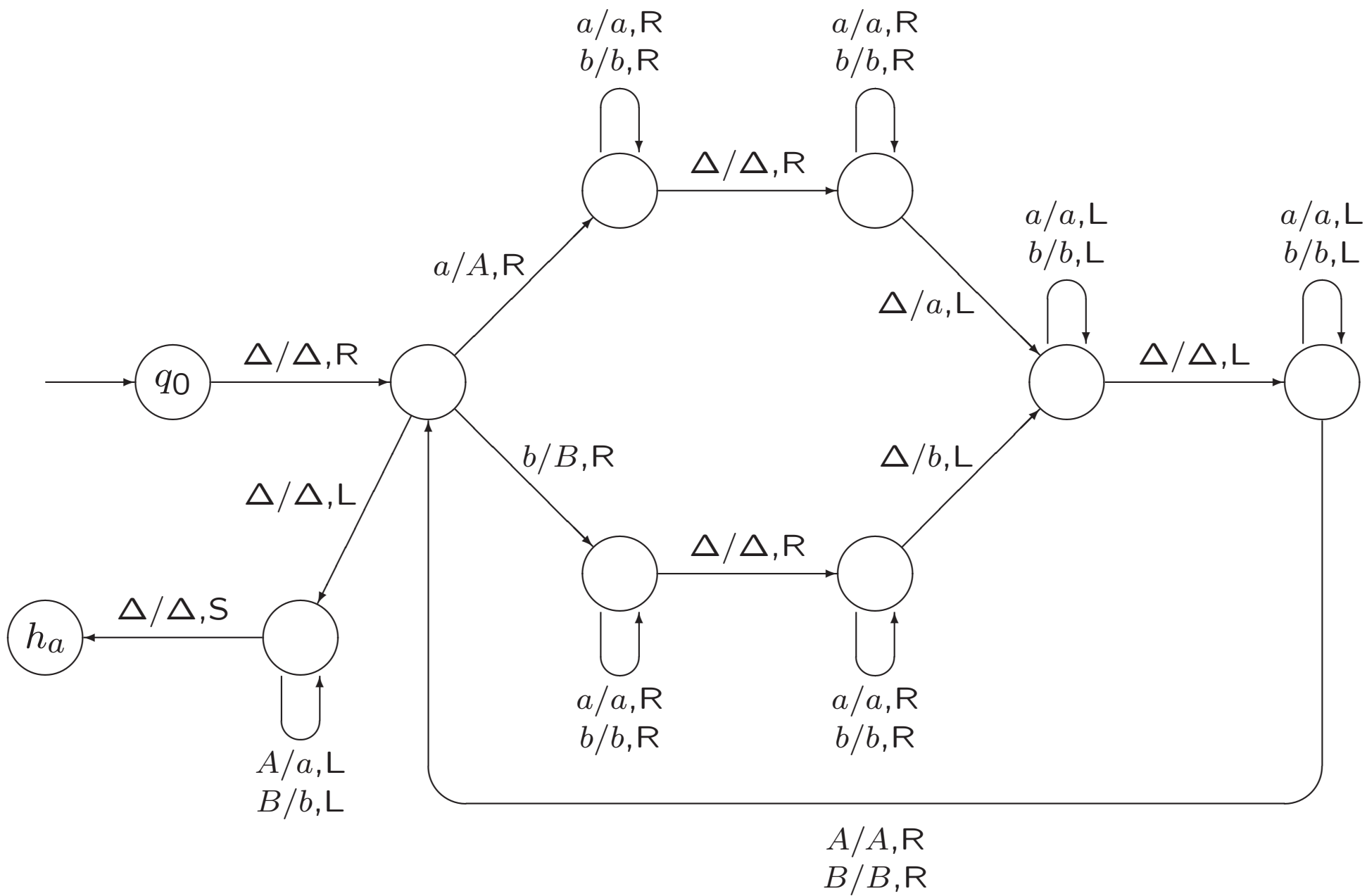
Example 7.10. The Reverse of a String

$$\underline{\triangle}\, a\, a\, b\, a\, b$$
$$\triangle\, A\, a\, b\, a\, b$$
$$\triangle\, A\, a\, b\, a\, A$$
$$\triangle\, B\, a\, b\, a\, A$$
$$\triangle\, B\, A\, b\, a\, A$$
$$\triangle\, B\, A\, b\, A\, A$$
$$\triangle\, B\, A\, b\, A\, A$$
$$\triangle\, B\, A\, B\, A\, A$$
$$\underline{\triangle}\, b\, a\, b\, a\, a$$

Example 7.24. Comparing Two Strings

Equal: accept $\underline{\triangle} x \triangle y$ if $x = y$,
and reject if $x \neq y$

An exercise from exercise class 2

Exercise 7.17.

For each case below, draw a TM that computes the indicated function.

e. $E : \{a, b\}^* \times \{a, b\}^* \to \{0, 1\}$
  defined by $E(x, y) = 1$ if $x = y$, $\qquad E(x, y) = 0$ otherwise.

Example 7.25. Accepting the Language of . . .

$$Copy \rightarrow NB \rightarrow R \rightarrow PB \rightarrow Equal$$

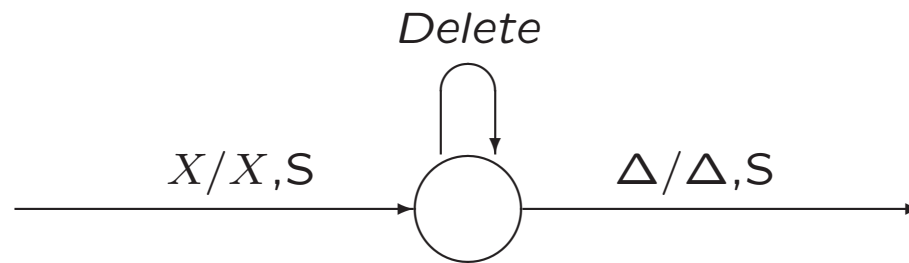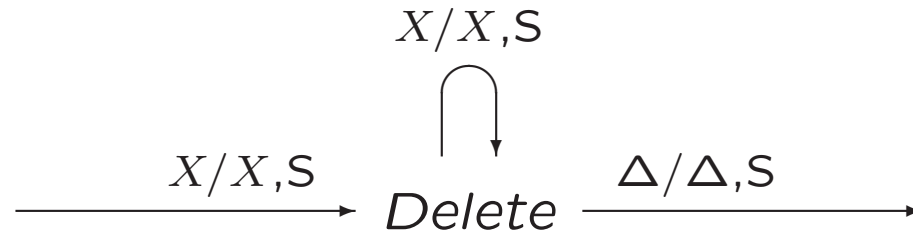**Example 7.25.** Accepting the Language of Palindromes

$$Copy \rightarrow NB \rightarrow R \rightarrow PB \rightarrow Equal$$
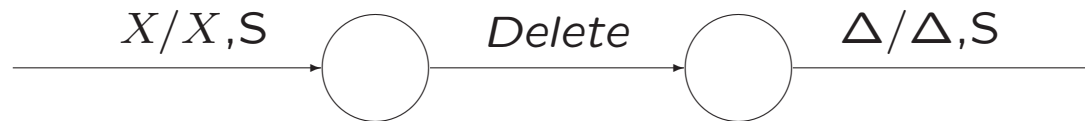
Example 7.21. Erasing the Tape

From the current position to the right

Many notations for composition

Incorrect notations for DeleteString:

$$X/X\text{,S}$$

$$X/X\text{,S} \qquad \text{Delete} \qquad \Delta/\Delta\text{,S}$$

Delete

$$X/X\text{,S} \qquad \bigcirc \qquad \Delta/\Delta\text{,S}$$

Not preferred notation:

$$X/X\text{,S} \qquad \bigcirc \qquad \text{Delete} \qquad \bigcirc \qquad \Delta/\Delta\text{,S}$$

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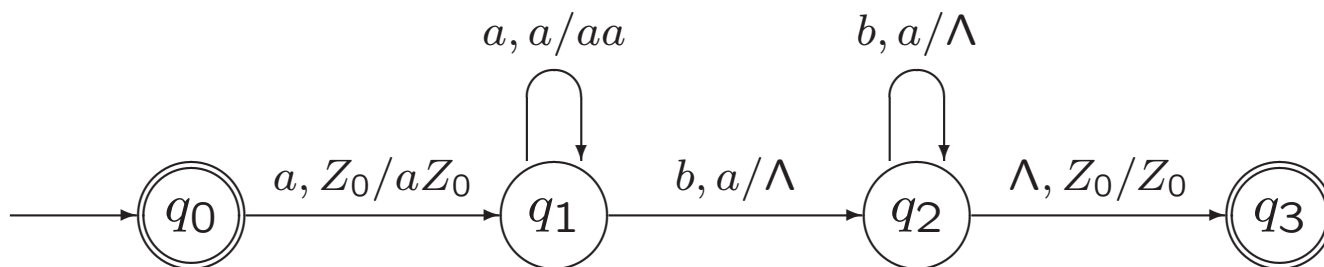
7.5. Multitape Turing Machines

Example 5.3. A PDA Accepting the Language *AnBn*

$$AnBn = \{a^i b^i \mid i \geq 0\}$$

Part of a slide from exercise class 1

Exercise 7.4.

For each of the following languages, draw a transition diagram for a Turing machine that accepts that language.

a. $AnBn = \{a^i b^i \mid i \geq 0\}$

With two tapes...

Exercise 7.4.

For each of the following languages, draw a transition diagram for a Turing machine that accepts that language.

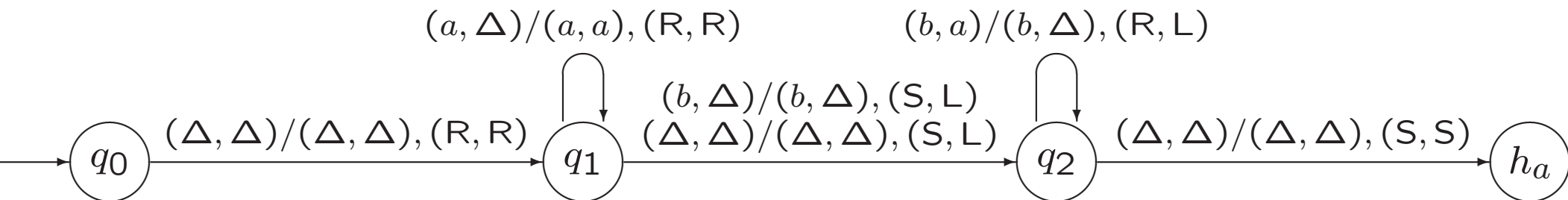**a.** $AnBn = \{a^i b^i \mid i \geq 0\}$

With two tapes:

$(a, \triangle)/(a, a), (\mathsf{R}, \mathsf{R})$ $(b, a)/(b, \triangle), (\mathsf{R}, \mathsf{L})$

$(b, \triangle)/(b, \triangle), (\mathsf{S}, \mathsf{L})$

$q_0 \xrightarrow{(\triangle, \triangle)/(\triangle, \triangle), (\mathsf{R}, \mathsf{R})} q_1 \xrightarrow{(\triangle, \triangle)/(\triangle, \triangle), (\mathsf{S}, \mathsf{L})} q_2 \xrightarrow{(\triangle, \triangle)/(\triangle, \triangle), (\mathsf{S}, \mathsf{S})} h_a$

Exercise 7.4.

For each of the following languages, draw a transition diagram for a Turing machine that accepts that language.

a. $AnBn = \{a^i b^i \mid i \geq 0\}$

We could also use the portion of the tape to the right of the input, to simulate the stack of a deterministic pushdown automaton (works for any deterministic PDA!)

Example 7.24. Comparing Two Strings

Equal: accept $\underline{\triangle} x \triangle y$ if $x = y$,
and reject if $x \neq y$

2-tape TM. . .

Multi-tape TM

- initial configuration. . .

- to compute a function. . .

Theorem 7.26. (informal)

For every 2-tape TM T, there is an ordinary 1-tape TM T_1, which for every input x,

- simulates the computation of T for x,

- accepts (rejects) x, if and only if T accepts (rejects) x,

- on acceptance, leaves the same output on its tape as T leaves on its first tape.

Proof idea, for x = aabb...

The proof of this result does not have to be known for the exam.

Corollary 7.27.

Every language that is accepted by a 2-tape TM can be accepted by an ordinary 1-tape TM,
and every function that is computed by a 2-tape TM can be computed by an ordinary TM.

This generalizes to k-tape TMs for $k \geq 3$.

7.7. Nondeterministic Turing Machines

A slide from lecture 2

Definition 7.1. Turing machines

A Turing machine (TM) is a 5-tuple $T = (Q, \Sigma, \Gamma, q_0, \delta)$, where

Q is a finite set of states. The two *halt* states h_a and h_r are not elements of Q.

Σ, the input alphabet, and Γ, the tape alphabet, are both finite sets, with $\Sigma \subseteq \Gamma$. The *blank* symbol Δ is not an element of Γ.

q_0, the initial state, is an element of Q.

δ is the transition function:

$$\delta : Q \times (\Gamma \cup \{\Delta\}) \rightarrow (Q \cup \{h_a, h_r\}) \times (\Gamma \cup \{\Delta\}) \times \{R, L, S\}$$

Nondeterministic Turing machine.

There may be more than one move for a state-symbol pair.

Same notation:

$$wpax \vdash_T yqbz \qquad wpax \vdash_T^* yqbz$$

A string x is accepted by T if

$$q_0 \Delta x \vdash_T^* w h_a y$$

for some strings $w, y \in (\Gamma \cup \{\Delta\})^*$.

NTM useful for accepting languages, for producing output, but not for computing function.

Example 7.28. The Set of Composite Natural Numbers.

Use *G2*

Example 7.28. The Set of Composite Natural Numbers.

$$NB \rightarrow G2 \rightarrow NB \rightarrow G2 \rightarrow PB \rightarrow M \rightarrow PB \rightarrow Equal$$

Take $x = 1^{15}$