

Foundations of Computer Science

Fundamentele Informatica 1

Hendrik Jan Hoogeboom
Jeannette de Graaf

Bachelor Informatica (& specialisaties)
Universiteit Leiden

Najaar 2020



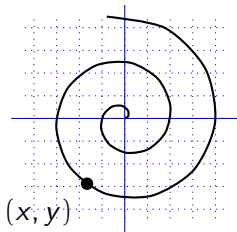
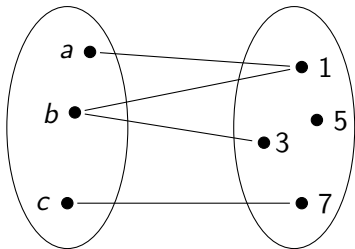
**Universiteit
Leiden**

Leiden Institute of
Advanced Computer Science

Hoofdstuk 2

Relaties

- 2 Relaties
 - Cartesisch product
 - Representaties
 - Eigenschappen
 - Compositie
 - Relaties “in”
 - Afsluiting
 - Partiële ordening
 - Equivalentierelatie



When two objects, qualities, classes, or attributes, viewed together by the mind, are seen under some connexion, that connexion is called a relation.

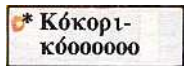
On the Syllogism, No. III, and on Logic in general, Transactions of the Cambridge Philosophical Society, 1858



Madurai, India 1806 – 1871 London
[wikipedia](#)

2 Relaties

- Cartesisch product
- Representaties
- Eigenschappen
- Compositie
- Relaties “in”
- Afsluiting
- Partiële ordening
- Equivalentierelatie



(geordend) *paar* (x, y)

$$(3, 5) \neq (5, 3) \quad \{3, 5\} = \{5, 3\}$$

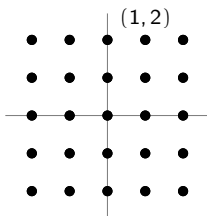
verzamelingen A, B

(Cartesisch) *product* $A \times B \quad \{ (x, y) \mid x \in A, y \in B \}$

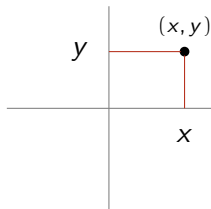
$$\{a, b, c\} \times \{1, 3, 5, 7\}$$

$(a,1)$	$(a,3)$	$(a,5)$	$(a,7)$
$(b,1)$	$(b,3)$	$(b,5)$	$(b,7)$
$(c,1)$	$(c,3)$	$(c,5)$	$(c,7)$

$$\mathbb{Z}^2 = \mathbb{Z} \times \mathbb{Z}$$



$$\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$$



Def. 2.1

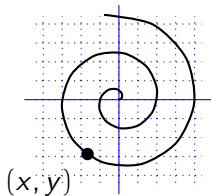
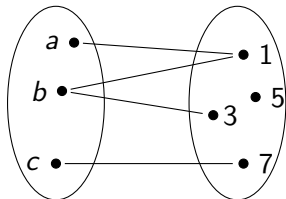
(binaire) *relatie* $R \subseteq A \times B$ “van A naar B ”

$R \subseteq A \times A$ “in A ”

$x R y$ $(x, y) \in R$ $a R b$

$\{(a, 1), (b, 1), (b, 3), (c, 7)\} \subseteq \{a, b, c\} \times \{1, 3, 5, 7\}$

$\{(r \cdot \cos(r), r \cdot \sin(r)) \mid r \geq 0\} \subseteq \mathbb{R} \times \mathbb{R}$



lijnen (in het vlak)

$$\ell \parallel m \text{ (parallel)} \quad \ell \perp m \text{ (loodrecht)}$$

getallen $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$

$$x = y \quad x \leq y \quad x < y \text{ (kleiner/gelijk)}$$

verzamelingen $\mathcal{P}(U)$ collectie \mathcal{C}

$$A \subseteq B \text{ (deelverzameling)} \quad A \cap B = \emptyset \text{ (disjunct)}$$

figuren (in het vlak)

$$\triangle ABC \cong \triangle A'B'C' \text{ (congruent)}$$

(gehele) getallen \mathbb{N}

$$x \mid y \text{ (deler)}$$

element vs verzameling

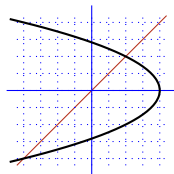
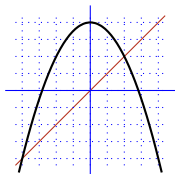
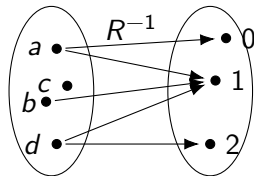
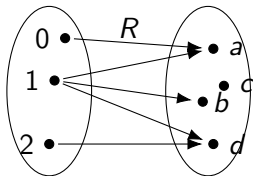
$$x \in A$$

$$R \subseteq A \times B$$

$$\text{inverse } R^{-1} \quad \{(y, x) \mid x R y\} \subseteq B \times A.$$

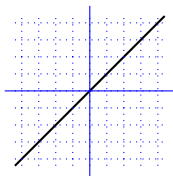
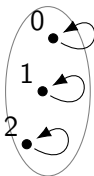
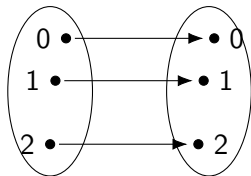
$$x \leq y \quad y \geq x$$

$$x \in A \quad A \ni x$$



identiteit $\text{id}_A = \{(a, a) \mid a \in A\} \subseteq A \times A$

gelijkheid, diagonaal $=_A \Delta_A$



1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1

$$R, S \subseteq A \times B$$

$$R \cup S \quad R \cap S \quad R^c \quad \subseteq A \times B$$

$x R^c y$ y desda $x \not R y$

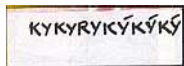
$$“<” \cup “=” = “\leq” \quad (?)$$

$$R_{<} \cup R_{=} = R_{\leq}$$

$$R_{\subseteq} \cap R_{\supseteq} = R_{=}$$

2 Relaties

- Cartesisch product
- Representaties
- Eigenschappen
- Compositie
- Relaties “in”
- Afsluiting
- Partiële ordening
- Equivalentierelatie



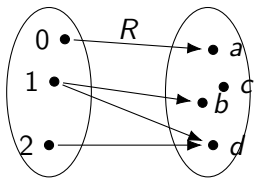
$$\{ (0, a), (1, b), (1, d), (2, d) \}$$

matrix

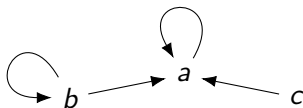
van \ naar	a	b	c	d
0	×			
1		×		×
2				×

	a	b	c	d
0	1	0	0	0
1	0	1	0	1
2	0	0	0	1

pijldiagram



gerichte graaf

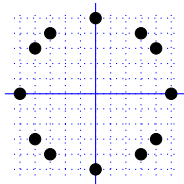
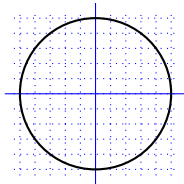
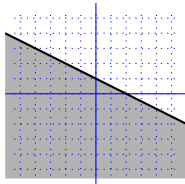
$$\{(a, a), (b, a), (b, b), (c, a)\}$$


grafiek 'nette' figuur

halfvlak $\{ (x, y) \in \mathbb{R} \times \mathbb{R} \mid x + 2y \leq 2 \} \subseteq \mathbb{R} \times \mathbb{R}$

cirkel $\{ (x, y) \in \mathbb{R} \times \mathbb{R} \mid x^2 + y^2 = 25 \}$

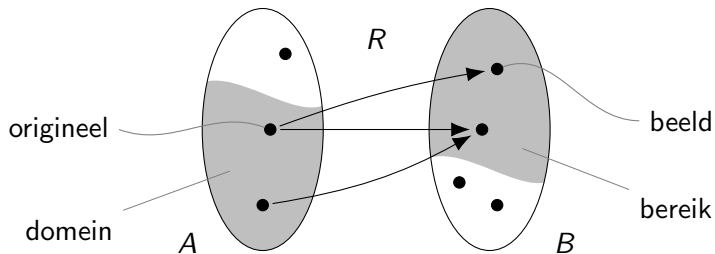
discreet $\{ (x, y) \in \mathbb{Z} \times \mathbb{Z} \mid x^2 + y^2 = 25 \}$



$$R \subseteq A \times B$$

domein $\text{dom}(R) = \{ x \in A \mid x R y \text{ voor zekere } y \in B \}$

bereik $\text{ran}(R) = \{ y \in B \mid x R y \text{ voor zekere } x \in A \}$ *range*



multidimensionaal

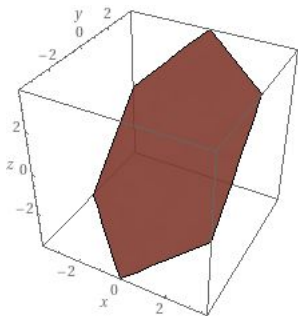
n-tupel

$$R \subseteq A_1 \times A_2 \times \cdots \times A_n \quad (a_1, a_2, \dots, a_n)$$

studentnr \times cursuscode \times cijfer

stnr	cuco	cf	stnr	cuco	cf
8303	M250	7	9594	T250	6
8303	T350	8	9352	U161	9
4722	B140	7	2592	A470	8
4722	S570	10	2592	M350	9
4722	T480	9	2592	V400	6

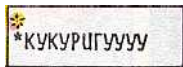
$$\text{Plus} = \{ (x, y, z) \in \mathbb{R}^3 \mid x + y = z \}$$

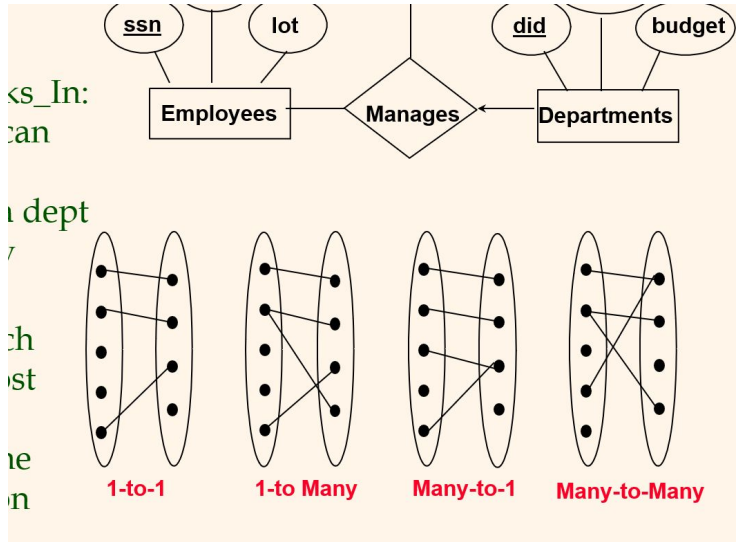


wolframalpha.com

2 Relaties

- Cartesisch product
- Representaties
- **Eigenschappen**
- Compositie
- Relaties “in”
- Afsluiting
- Partiële ordening
- Equivalentierelatie





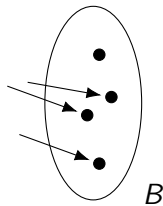
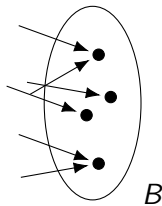
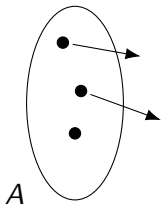
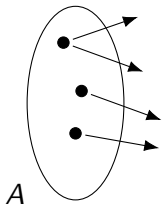
$$R \subseteq A \times B$$

vanuit A overal

- minimaal één pijl *totaal*
- maximaal één pijl *functioneel*

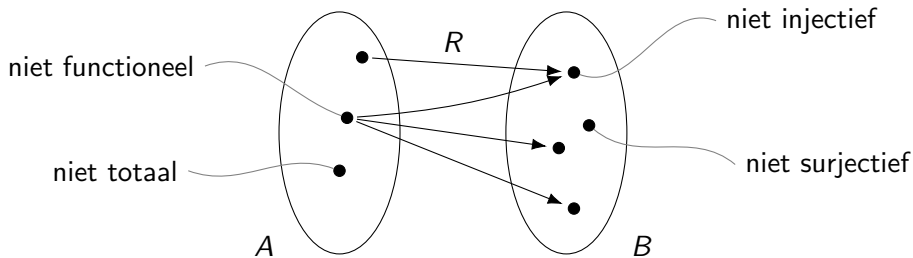
naar B overal

- minimaal één pijl *surjectief* (of 'op')
- maximaal één pijl *injectief* (of één-één)

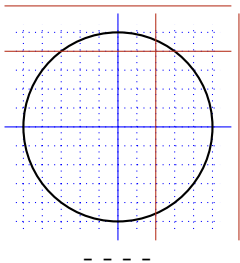


$$R \subseteq A \times B$$

- *totaal* $\text{dom}(R) = A$
- *surjectief* $\text{ran}(R) = B$ (of 'op')
- *functioneel* uit $x R y$ en $x R z$ volgt dat $y = z$
- *injectief* uit $x R z$ en $y R z$ volgt dat $x = y$ (of *één-één*)

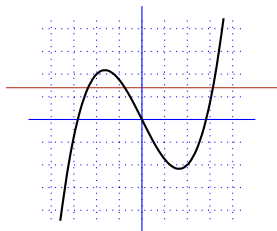


totaal surjectief functioneel injectief



	1	2	3	4
a	1	0	0	0
b	0	1	1	0
c	1	0	1	0

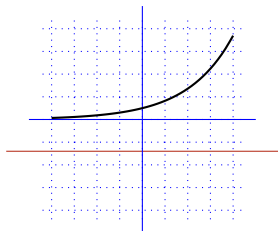
T - - -



TSF -

	a	b	c
1	1	0	0
2	0	1	0
3	0	0	0
4	0	0	1

- SFI



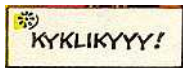
T - FI

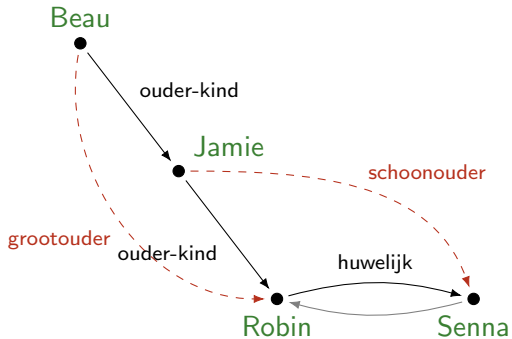
	a	b	c
a	1	0	0
b	0	0	1
c	0	1	0

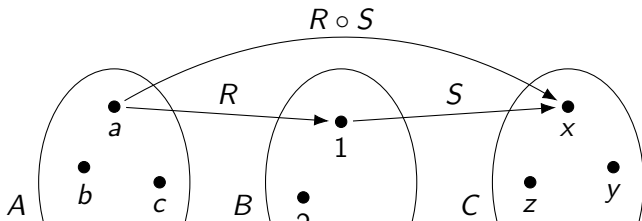
TSFI

2 Relaties

- Cartesisch product
- Representaties
- Eigenschappen
- **Compositie**
- Relaties “in”
- Afsluiting
- Partiële ordening
- Equivalentierelatie







$$R \subseteq A \times B \quad S \subseteq B \times C$$

$$a R 1 \quad 1 S x$$

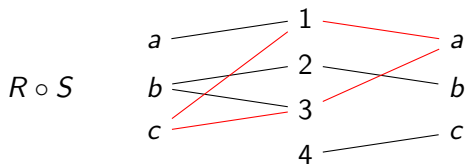
$$a (R \circ S) x$$

samenstelling (compositie) van R en S $R \circ S \subseteq A \times C$

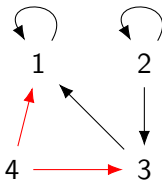
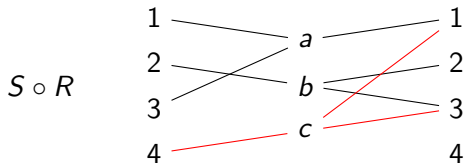
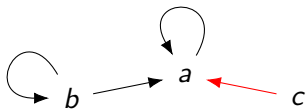
$$\{ (x, z) \in A \times C \mid \text{er is een } y \in B \text{ met } (x, y) \in R \text{ en } (y, z) \in S \}$$

$$R = \{(a, 1), (b, 2), (b, 3), (c, 1), (c, 3)\} \subseteq \{a, b, c\} \times \{1, 2, 3, 4\}$$

$$S = \{(1, a), (2, b), (3, a), (4, c)\} \subseteq \{1, 2, 3, 4\} \times \{a, b, c\}$$



$$R \circ S = \{(a, a), (b, a), (b, b), (c, a)\}$$



relaties

$$\overrightarrow{x R y}$$

$$x (\overrightarrow{R \circ S}) y$$

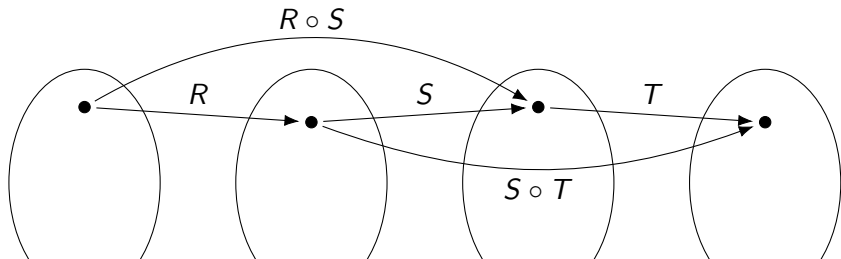
functies


$$y = \overleftarrow{f}(x)$$

$$y = \overleftarrow{g}(f(x)) = (\overleftarrow{g \circ f})(x)$$

Thm. 2.1

$$R \circ (S \circ T) = (R \circ S) \circ T$$



matrixvermenigvuldiging 

$$A = (a_{ik}) \quad B = (b_{kj})$$

$$c_{ij} = \sum_k a_{ik} \cdot b_{kj}$$

$$c_{ij} = \bigvee_k a_{ik} \wedge b_{kj}$$

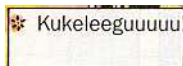
	1	2	3	4
a	1	0	0	0
b	0	1	1	0
c	1	0	1	0

	x	y	z
1	1	0	0
2	0	1	0
3	1	0	0
4	0	0	1

	x	y	z
a	1	0	0
b	1	1	0
c	1	0	0

2 Relaties

- Cartesisch product
- Representaties
- Eigenschappen
- Compositie
- **Relaties “in”**
- Afsluiting
- Partiële ordening
- Equivalentierelatie



lijnen (in het vlak)

$$\ell \parallel m \text{ (parallel)} \quad \ell \perp m \text{ (loodrecht)}$$

getallen $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$

$$x = y \quad x \leq y \quad x < y \text{ (kleiner/gelijk)}$$

verzamelingen $\mathcal{P}(U)$ collectie \mathcal{C}

$$A \subseteq B \text{ (deelverzameling)} \quad A \cap B = \emptyset \text{ (disjunct)}$$

figuren (in het vlak)

$$\triangle ABC \cong \triangle A'B'C' \text{ (congruent)}$$

positieve gehele getallen \mathbb{N}^+

$$x \mid y \text{ (deler)}$$

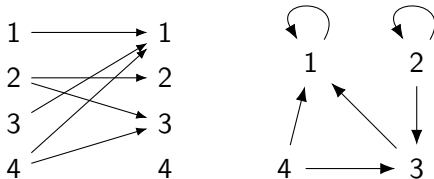
gehele getallen \mathbb{Z}

$$x \equiv y \text{ (zelfde rest, bij deling door } n)$$

$$R \subseteq A \times A$$

representatie: **gerichte graaf**

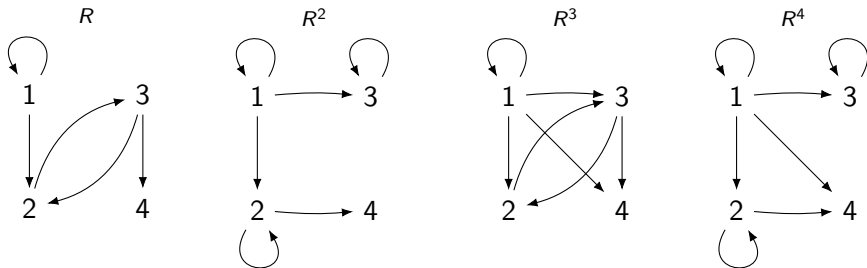
$$\{(1, 1), (2, 2), (2, 3), (3, 1), (4, 1), (4, 3)\}$$



$$R \subseteq A \times A$$

herhaald samenstellen

$$R^1 = R \quad R^2 = R \circ R \quad R^3 = R^2 \circ R \quad \dots$$



$$R = \{ (1, 1), (1, 2), (2, 3), (3, 2), (3, 4) \}$$

$$R^2 = \{ (1, 1), (1, 2), (1, 3), (2, 2), (2, 4), (3, 3) \}$$

$$R^3 = \{ (1, 1), (1, 2), (1, 3), (1, 4), (2, 3), (3, 2), (3, 4) \}$$

$$R^4 = \{ (1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 4), (3, 3) \}$$



equivalentie

reflexief

symmetrisch

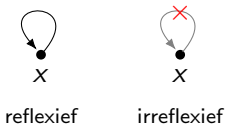
transitief

partiele ordening

reflexief

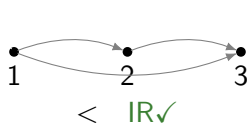
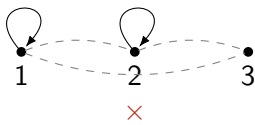
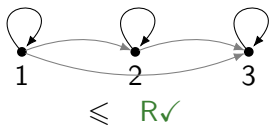
*anti*symmetrisch

transitief



$$R \subseteq A \times A$$

- *reflexief* $x R x$ voor elk element x van A ,
- *irreflexief* $x R x$ voor *geen enkele* $x \in A$



lijnen (in het vlak)

$$\ell \parallel m \text{ R}\checkmark \text{ (parallel)} \quad \ell \perp m \text{ R}\times \text{ IR}\checkmark \text{ (loodrecht)}$$

getallen $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$

$$x = y \text{ R}\checkmark \quad x \leq y \text{ R}\checkmark \quad x < y \text{ IR}\checkmark \text{ (kleiner/gelijk)}$$

verzamelingen $\mathcal{P}(U)$ collectie \mathcal{C}

$$A \subseteq B \text{ R}\checkmark \text{ (deelverzameling)} \quad A \cap B = \emptyset \text{ R}\times \text{ IR}\times \text{ (disjunct)}$$

figuren (in het vlak)

$$\triangle ABC \cong \triangle A'B'C' \text{ R}\checkmark \text{ (congruent)}$$

positieve gehele getallen \mathbb{N}^+

$$x \mid y \text{ R}\checkmark \text{ (deler)}$$

gehele getallen \mathbb{Z}

$$x \equiv y \text{ R}\checkmark \text{ (zelfde rest, bij deling door } n\text{)}$$

lijnen (in het vlak)

$l \parallel m$ $R\checkmark$ (parallel) $l \perp m$ $R \times IR\checkmark$ (loodrecht)

“not reflexive since no line is parallel to itself.” (? Schaum p.29)

verzamelingen $\mathcal{P}(U)$ collectie \mathcal{C}

$A \subseteq B$ $R\checkmark$ (deelverzameling) $A \cap B = \emptyset$ $R \times IR \times$ (disjunct)

$\{a, b\} \cap \{a, b\} \neq \emptyset$ $\emptyset \cap \emptyset = \emptyset$ (!)

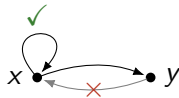
positieve gehele getallen \mathbb{N}^+

$x | y$ $R\checkmark$ (deler)

$x = 1 \cdot x$



symmetrisch



anti-symmetrisch

$$R \subseteq A \times A$$

- *symmetrisch* uit $x R y$ volgt dat $y R x$
- *anti-symmetrisch* uit $x R y$ en $y R x$ volgt dat $x = y$

anti-symmetrisch “geen ... tussen *verschillende*”

lijnen (in het vlak)

$$\ell \parallel m \text{ S✓ (parallel)} \quad \ell \perp m \text{ S✓ (loodrecht)}$$

getallen $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$

$$x = y \text{ S AS✓ (!)} \quad x \leq y \text{ AS✓} \quad x < y \text{ AS✓ (!) (kleiner/gelijk)}$$

verzamelingen $\mathcal{P}(U)$ collectie \mathcal{C}

$$A \subseteq B \text{ AS✓ (deelverzameling)} \quad A \cap B = \emptyset \text{ S✓ (disjunct)}$$

figuren (in het vlak)

$$\triangle ABC \cong \triangle A'B'C' \text{ S✓ (congruent)}$$

positieve gehele getallen \mathbb{N}^+

$$x \mid y \text{ AS✓ (deler)}$$

gehele getallen \mathbb{Z}

$$x \equiv y \text{ S✓ (zelfde rest, bij deling door } n\text{)}$$

getallen $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$

$$x = y \text{ AS}\checkmark (!)$$

$$x \leq y \text{ AS}\checkmark \quad \text{als } x \leq y \text{ en } y \leq x \text{ dan } x = y$$

$$x < y \text{ AS}\checkmark (!) \quad \text{als } \underbrace{x < y \text{ en } y < x}_{\text{kan niet}} \text{ dan } \dots$$

verzamelingen $\mathcal{P}(U)$ collectie \mathcal{C}

$$A \subseteq B \text{ AS}\checkmark \quad \text{als } A \subseteq B \text{ en } B \subseteq A \text{ dan } A = B$$

$$A \subset B \text{ AS}\checkmark \quad \text{"kan niet"}$$

positieve gehele getallen \mathbb{N}^+

$$x \mid y \text{ AS}\checkmark \text{ (deler)}$$

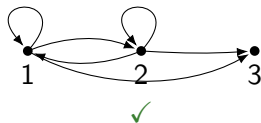
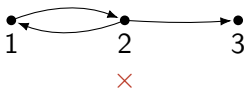
$$x \mid y \quad \text{dan } x \leq y$$

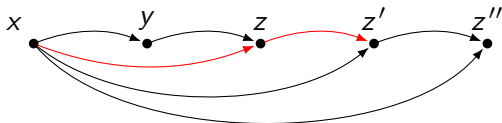
$$y \mid x \quad \text{dan } y \leq x$$



$$R \subseteq A \times A$$

- *transitief* uit $x R y$ en $y R z$ volgt dat $x R z$





$$R \subseteq A \times A$$

- *transitief* uit $x R y$ en $y R z$ volgt dat $x R z$

lijnen (in het vlak)

$$\ell \parallel m \text{ T}\checkmark \text{ (parallel)} \quad \ell \perp m \times \text{ (loodrecht)}$$

getallen $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$

$$x = y \text{ T}\checkmark \quad x \leq y \text{ T}\checkmark \quad x < y \text{ T}\checkmark \text{ (kleiner/gelijk)}$$

verzamelingen $\mathcal{P}(U)$ collectie \mathcal{C}

$$A \subseteq B \text{ T}\checkmark \text{ (deelverzameling)} \quad A \cap B = \emptyset \times \text{ (disjunct)}$$

figuren (in het vlak)

$$\triangle ABC \cong \triangle A'B'C' \text{ T}\checkmark \text{ (congruent)}$$

positieve gehele getallen \mathbb{N}^+

$$x \mid y \text{ T}\checkmark \text{ (deler)}$$

gehele getallen \mathbb{Z}

$$x \equiv y \text{ T}\checkmark \text{ (zelfde rest, bij deling door } n\text{)}$$

verzamelingen $\mathcal{P}(U)$ collectie \mathcal{C}

$$A \cap B = \emptyset \quad \text{T} \times \quad (\text{deler})$$

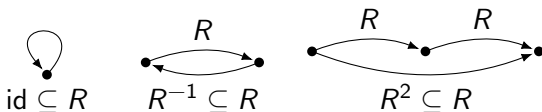
$$\{1\} \cap \{2\} = \emptyset, \text{ en } \{2\} \cap \{1\} = \emptyset \text{ maar } \{1\} \cap \{1\} \neq \emptyset$$

positieve gehele getallen \mathbb{N}^+

$$x \mid y \quad \text{T} \checkmark \quad (\text{deler})$$

$$x \mid y \text{ dwz } y = k \cdot x$$

$$y \mid z \text{ dwz } z = m \cdot y \quad z = mk \cdot x \quad \text{dus } x \mid z$$



$R \subseteq V \times V$ is

- ① reflexief desda $\text{id}_V \subseteq R$
- ② irreflexief desda $\text{id}_V \cap R = \emptyset$
- ③ symmetrisch desda $R^{-1} \subseteq R$
- ④ antisymmetrisch desda $R^{-1} \cap R \subseteq \text{id}_V$
- ⑤ transitief desda $R^2 \subseteq R$

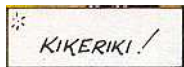
Thm. 2.2

$R \subseteq V \times V$ is

transitief desda $R^n \subseteq R$ voor alle $n \geq 1$

2 Relaties

- Cartesisch product
- Representaties
- Eigenschappen
- Compositie
- Relaties “in”
- **Afsluiting**
- Partiële ordening
- Equivalentierelatie



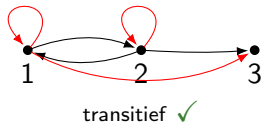
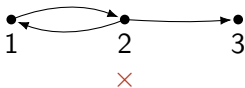
P relatie eigenschap

(ir)reflexief, (anti)symmetrisch, transitief, ...

$R \subseteq A \times A$ P -afsluiting

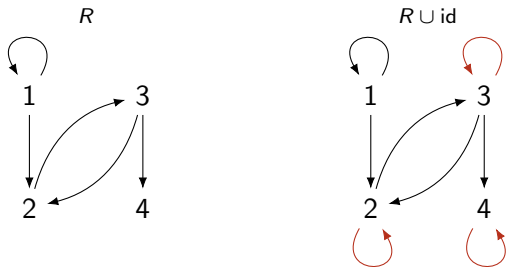
– $R \subseteq P(R) \subseteq A \times A$ met eigenschap P .

– kleinste



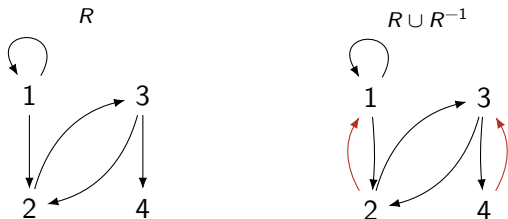
– is er wel een $P(R)$?

– is er een (unieke) kleinste?



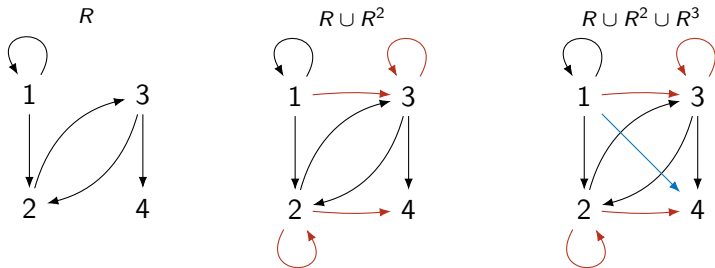
$$R = \{ (1, 1), (1, 2), (2, 3), (3, 2), (3, 4) \}$$

$$\text{id}_{\{1,2,3,4\}} = \{ (1, 1), (2, 2), (3, 3), (4, 4) \}$$



$$R = \{ (1, 1), (1, 2), (2, 3), (3, 2), (3, 4) \}$$

$$R^{-1} = \{ (1, 1), (2, 1), (3, 2), (3, 2), (4, 3) \}$$



$$R = \{ (1, 1), (1, 2), (2, 3), (3, 2), (3, 4) \}$$

$$R^2 = \{ (1, 1), (1, 2), (1, 3), (2, 2), (2, 4), (3, 3), (3, 4) \}$$

$$R^3 = \{ (1, 1), (1, 2), (1, 3), (1, 4), (2, 3), (3, 2), (3, 4) \}$$

$$R^4 = \{ (1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 4), (3, 3) \}$$

$$R \subseteq A \times A$$

Thm. 2.3

$$\text{reflexief}(R) = R \cup \text{id}_A$$

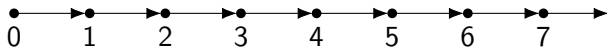
$$\text{symmetrisch}(R) = R \cup R^{-1}$$

$$R^1 = R, R^2 = R^1 \circ R, R^3 = R^2 \circ R, \dots, R^{n+1} = R^n \circ R,$$

$$R^+ = \bigcup_{n \geq 1} R^n \quad \text{“een, twee, of meer stappen”}$$

Thm. 2.4

$$\text{transitief}(R) = R^+$$



" $+1$ " $\subseteq \mathbb{N} \times \mathbb{N}$

$$R = \{ (n, n+1) \mid n \in \mathbb{N} \}$$

$$R^k = \{ (n, n+k) \mid n \in \mathbb{N} \} \quad k \text{ vast}$$

$$R^+ = \{ (n, m) \mid n < m \} \quad \text{oneindige vereniging}$$

$$R \subseteq A \times A$$

$$R^0 = \text{id}_A, R^1 = R, R^2 = R^1 \circ R, \dots, R^{n+1} = R^n \circ R,$$

$$R^+ = \bigcup_{n \geq 1} R^n \quad \text{transitief}$$

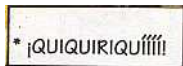
“een, twee, of meer stappen”

$$R^* = \bigcup_{n \geq 0} R^n \quad \text{reflexief én transitief}$$

“nul, een, twee, of meer stappen”

2 Relaties

- Cartesisch product
- Representaties
- Eigenschappen
- Compositie
- Relaties “in”
- Afsluiting
- **Partiële ordening**
- Equivalentierelatie





partiële ordening



“past in”

reflexief, **antisymmetrisch**, transitief

equivalentierelatie



“zelfde (kleur)”

reflexief, **symmetrisch**, transitief

voorbeelden (partiële ordening, equivalentie)

lijnen (in het vlak)

$$\ell \parallel m \text{ EqRel (parallel)}$$

getallen $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$

$$x \leq y \text{ PartOrd}$$

$$x = y \text{ EqRel}$$

verzamelingen $\mathcal{P}(U)$ collectie \mathcal{C}

$$A \subseteq B \text{ PartOrd (deelverzameling)}$$

figuren (in het vlak)

$$\triangle ABC \cong \triangle A'B'C' \text{ EqRel (congruent)}$$

positieve gehele getallen \mathbb{N}^+

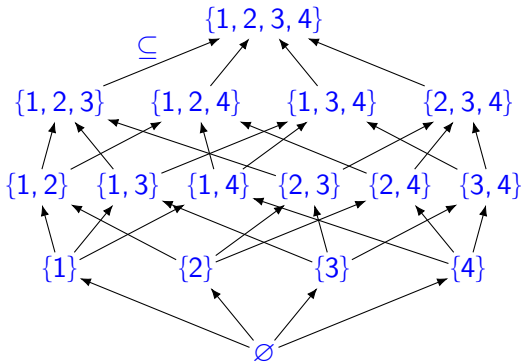
$$x \mid y \text{ PartOrd (deler)}$$

gehele getallen \mathbb{Z}

$$x \equiv y \text{ EqRel (zelfde rest, bij deling door } n)$$

$R \subseteq A \times A$ partiële ordening reflexief, antisymmetrisch, transitief

inclusie \subseteq in $\mathcal{P}(\{1, 2, 3, 4\})$

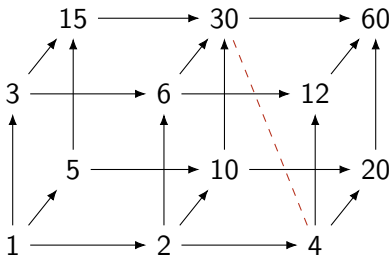


Hasse diagram \pm reflexief \pm transitief

$D = \{1, 2, 3, 4, 5, 6, 10, 15, 20, 30, 60\}$ delers van 60

deler $|$ in D

partieel



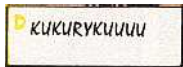
kleiner(gelijk) \leq in D

totaal

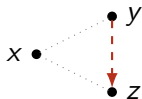
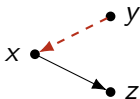
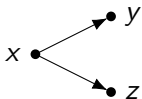
1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6 \rightarrow 10 \rightarrow 12 \rightarrow 15 \rightarrow 20 \rightarrow 30 \rightarrow 60

2 Relaties

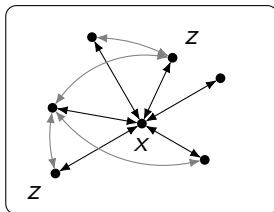
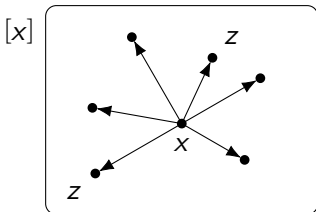
- Cartesisch product
- Representaties
- Eigenschappen
- Compositie
- Relaties “in”
- Afsluiting
- Partiële ordening
- **Equivalentierelatie**



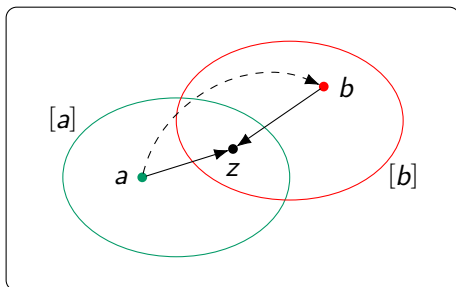
$R \subseteq A \times A$ **equivalentierelatie** reflexief, symmetrisch, transitief
 als $x R y$ en $x R z$ dan $y R z$



equivalentie klasse $[x] = \{ z \mid x R z \}$



$R \subseteq A \times A$ **equivalentierelatie** reflexief, symmetrisch, transitief



Thm. 2.6

- ① $a \in [a]$
- ② $[a] = [b]$ desda $a R b$
- ③ $[a] \neq [b]$ desda $[a] \cap [b] = \emptyset$

- in \mathbb{Z} $x - y$ deelbaar door 5 $x \equiv y \pmod{5}$

-11	-6	-1	4	9	14	19	...
...	-7	-2	3	8	13	18	...
...	-8	-3	2	7	12	17	...
...	-9	-4	1	6	11	16	...
...	-10	-5	0	5	10	15	...

eigenschap: rest (bij deling door 5)

- parallele lijnen $\ell \parallel m$ richtingscoëfficiënt
- congruentie $\triangle ABC \cong \triangle A'B'C'$ grootte & vorm

- Cartesisch product
- relatie
- inverse
- Boolese operaties
- totaal, functioneel, surjectief, injectief
- compositie (samenstelling)
- ir/reflexief
- anti/symmetrie
- transitief

